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SPECTRAL ANALYSIS USING A METHOD OF SEQUENTIAL TRANSFORMS

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ABSTRACT

We are exploring the use of acoustic resonant techniques to identify defective components by monitoring the fundamental modes of vibration that arise following an impulse load. We employ the spectrum of the filtered data as a feature for use in automated statistical pattern recognition. The feature provides a measure of the severity of the defect using the acoustic signature of a defect free component as a baseline. Defects change the signature by shifting or splitting the frequencies and introducing beats. Identifying these changes in the spectrum relative to baseline data is difficult because of the large number of natural frequencies associated with complex geometries and the presence of noise in the data. In the case of real-time health monitoring, obtaining a representative spectrum is challenging since averaging cannot be employed without the use of multiple transducers. We have developed a new approach to obtain a smoother spectrum by analyzing sequential data using an approach similar to Rayleigh's quotient method for eigenvalue problems and super-resolution techniques. The resulting spectrum enhances the natural modes of vibration and provides a better feature for automated classification techniques. The numerical implementation of the algorithm requires modest computational resources. The algorithm for generating the Rayleigh quotient has been successfully tested on numerically generated waveforms and experimentally acquired data. In all cases, there was a significant reduction in the inherent noise and the numerically generated noise from digital filtering.

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BACKGROUND

The derivation of Rayleigh's quotient is outlined below to provide insight into how it can be obtained as sequential transforms. These transforms can then be used to simplify the numerical techniques so the approach can be implemented using modest computational resources.

Consider a signal G_i ,

$$G_i = A_i \exp(-2\pi j f_0 t_i) = A_i \exp(-2\pi j f_0 \Delta t i), \quad i = 0, N-1 \tag{1}$$

where A_i is the amplitude, f_0 is a single frequency, and t is a time vector. The Discrete Fourier Transform (DFT) of G_i is:

$$H_l = \frac{1}{N}\sum_{i=0}^{i=N-1} G_i \exp(2\pi j i \frac{l}{N}) = \frac{1}{N}\sum_{i=0}^{N-1} A_i \exp(2\pi j i(\frac{l}{N} - f_0 \Delta t)), \quad l = 0,..,N-1 \tag{2}$$

Let $\alpha = -2\pi f_0 \Delta t, \quad \beta = -\frac{2\pi l}{N}, \quad a = \exp(j\alpha), \quad b = \exp(j\beta) \quad b = b(l), \ \theta = \frac{\alpha - \beta}{2}$

Let A_i = 1, and sum the series to give H_i as:

$$\left|H_l(N)\right|^2 = \left\{\frac{\sin(N\theta)}{N\sin(\theta)}\right\}^2 \tag{3}$$

Equations (2,3) are the usual Fourier transform of G_i . Equation (3) can also be obtained by constructing a covariance matrix using super-resolution methods [4]. A set of vectors is defined, using a step-wise approach, as:

$$\begin{aligned}
&x_1 = (a^{N-1} \ a^{N-2} \cdots 1) \\
&x_2 = (a^{N-2} \ a^{N-3} \cdots 0), \quad etc \\
&\vdots \\
&x_N = (1\,0\,0 \cdots\cdots 0) \\
&B = (b^{N-1} \ b^{N-2} \cdots 1)^T
\end{aligned} \tag{4}$$

The Rayleigh's quotient, λ_N , is [5]:

$$\hat{x}_N = (x_N \ x_{N-1} \ x_{N-2} \cdots\cdots x_1)^T$$

$$\hat{x}_N B = (x_N B \quad x_{N-1} B \quad x_{N-2} B \quad \cdots x_1 B)^T \tag{5}$$

$$\lambda_N = \frac{(\hat{x}_N B)^T (\hat{x}_N B)}{B^T B}$$

$$Cov(\hat{x}_N) = (\hat{x}_N)^T (\hat{x}_N) \tag{6}$$

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Equation (6) is the covariance matrix of source vector and *B* is the test vector. Rayleigh's quotient provides an alternate to the Fast Fourier Transform (FFT), but requires a large matrix and is numerically intensive. Figure 1 shows MATLAB [5] code used to generate the quotient. The complexity can be reduced by analyzing the structure of the quotient.

```
function RALY1=Rayleigh_old(sig,t,freq_F)
vector1=sig;
NN=length(sig);
xx1=zeros(NN,NN);
for lag=0:NN-1
xx1(:,lag+1)=[zeros(1,lag), conj(vector1(1:NN-lag))]';
end
R1 = xx1'*xx1; % Correlation matrix
f0=freq_F;
for k=1:NN
S=exp(-2*pi*i*f0(k)*t);
RALY1(k)=(S*R1*S')/(S*S');
end
return
```

Figure 1 MATLAB code for generating the Rayleigh Quotient

.

It should be noted that in above definition λ_N has the structure of an eigenvalue but it is actually a transform of the original signal. It has an implicit transform variable $l = 0,..,N-1$. The following shows how λ_N can be given as the sum of the squares of sequential Fourier transform.

Consider only one term in (5):

$$\begin{aligned}
\lambda_1 &= \frac{(x_1B)^T(x_1B)}{B^TB} \\
\lambda_1 &= \frac{1}{B^TB}\left\{(\frac{a}{b})^{N-1} + (\frac{a}{b})^{N-2} + \cdots + 1\right\}^T \left\{(\frac{a}{b})^{N-1} + (\frac{a}{b})^{N-2} + \cdots + 1\right\} \\
&= \frac{1}{N}\frac{1}{a^{N-1}}\left\{\frac{a^N - b^N}{a-b}\right\}\frac{1}{b^{N-1}}\left\{\frac{a^N - b^N}{a-b}\right\} \\
&= \frac{1}{N}\frac{1}{(ab)^{N-1}}\left\{\frac{a^N - b^N}{a-b}\right\}^2 = \frac{1}{N}\frac{\sin^2 N\theta}{\sin^2\theta} = N\left|H_l\right|^2
\end{aligned} \tag{7}$$

Similarly,

$$\lambda_2 = \frac{(x_2B)^T(x_2B)}{B^TB} \tag{8}$$

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Considering the sum of all the terms and the identity $(a-b)^T(a-b) = 4\sin^2\theta$:

$$\lambda_N = \frac{b^{N-1}}{N(a-b)^T}((a-b)^T \cdots (a^N - b^N)^T)\frac{1}{b^{N-1}}\frac{1}{(a-b)}\begin{pmatrix} a-b \\ a^2-b^2 \\ \vdots \\ a^N - b^N \end{pmatrix}$$
$$= \frac{1}{N}\frac{1}{4\sin^2\theta}\left\{4\sin^2\theta + 4\sin^2 2\theta + 4\sin^2 3\theta + \cdots + 4\sin^2 N\theta\right\} \tag{9}$$

Using the summation formula we have:

$$\lambda_N = \sum_{k=1}^{N}\left\{\frac{\sin(k\theta)}{\sin\theta}\right\}^2 = \frac{2N+1}{4N\sin^2\theta}\left\{1 - \frac{\sin(2N+1)\theta}{(2N+1)\sin\theta}\right\} \tag{10}$$

Equation 10 is the Rayleigh Quotient transform of a signal with a single frequency f0 as compared to equation 2 and equation 3 which are Fourier transforms. For comparison equation 2 has been simplified for A_i=1 as:

$$|H_l| = \frac{SinN\theta}{N\sin\theta} \tag{11}$$

Equations (10) and (11) incorporate sinc functions and for the ranges involved, can be less than 1. Therefore, the Fourier transforms can be zero resulting in negative spikes. The Rayleigh quotients will never be zero.

For numerical implementation, the Rayleigh quotient can be represented as the sum:

$$\lambda_N = \frac{1}{N}\frac{1}{4\sin^2\theta}\left\{4\sin^2\theta + 4\sin^2 2\theta + 4\sin^2 3\theta + \cdots + 4\sin^2 N\theta\right\}$$
$$= \frac{1}{N}\left\{H^2{}_{lN} + H^2{}_{lN-1}\cdots H^2{}_{l0})\right\}$$
$$= N\left\{\frac{fft^2(x_1)}{N^2} + \frac{fft^2(x_2)}{N^2}\cdots\frac{fft^2(x_N)}{N^2}\right\} \tag{12}$$

where $|H_l(N-i)| = \dfrac{Sin(N-i)\theta}{N\sin\theta} \quad i = 1,2,3\cdots$

Figure 2 is the corresponding MATLAB code for generating the sequential transform which uses the FFT routine. The input is the time series signal and the output is Rayleigh quotient.

```
function Ys=Rayleigh(x)
tot=0;
x=fliplr(x);
for i=0:length(x)-1,tot=tot+abs(fft([x(i+1:end),zeros(1,i)])).^2;end
Ys=tot/length(x);
return
```

Figure 2 code for efficient generation of the Rayleigh Quotient.

RESULTS

The algorithm for generating the Rayleigh quotient was tested on numerically generated waveforms and experimentally acquired data. In all cases, the inherent noise and the noise generated numerically (i.e. sidelobes from numerical filters) was reduced significantly. Figure 3 shows the normalized FFT of a signal and figure 4 shows the transform obtained with Raleigh quotient. The signal, $0.5\sin(2\pi 1000t)+1.0\sin(2\pi 1500t)$, was generated numerically and corrupted with random numbers with a signal to noise ratio of -14 dB.

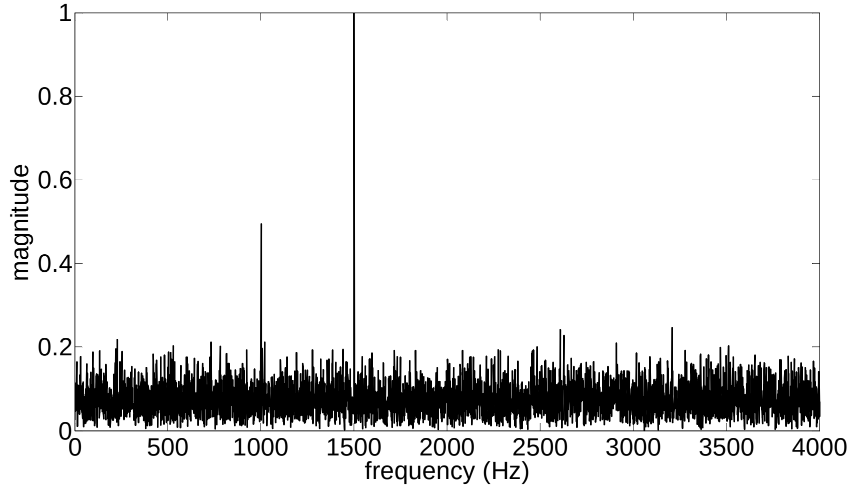

Figure 3. FFT of $0.5\sin(2\pi 1000t)+1.0\sin(2\pi 1500t)$

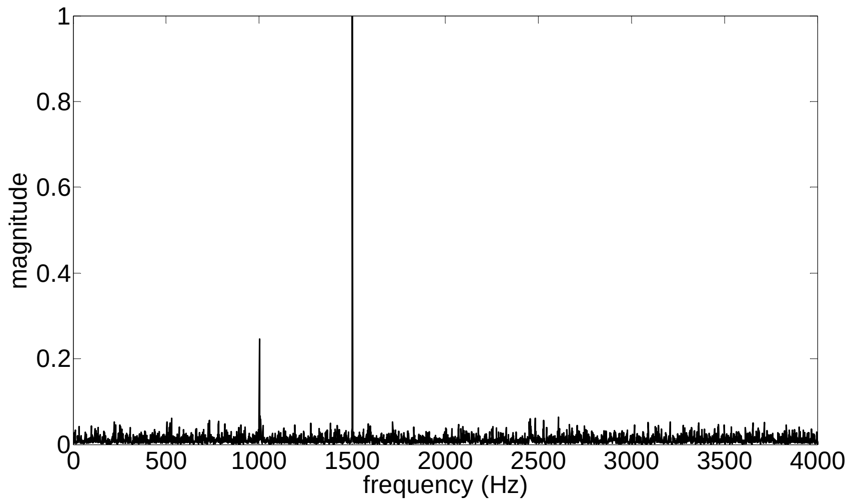

Figure 4. Rayleigh Quotient of $0.5\sin(2\pi 1000t)+1.0\sin(2\pi 1500t)$

We are currently employing a compact tensile specimen in a systematic study to quantify the propagation of a crack in terms of changes in the fundamental modes of vibration. A crack was induced in a tensile specimen and grown in regular increments. Raw vibration data was collected at each increment using two accelerometers mounted to the specimen, an instrumented hammer to generate the impulse load, and a spectrum analyzer to collect the data. Approximately 100 data sets from each accelerometer were collected for each specimen. Figure 5 shows the average of the FFT for second fundamental (9407 Hz, defect free) at each test increment. Results are similar for the first and third modes. Figure 6 shows the second fundamental computed for a single data set using the FFT and method of sequential transforms. The data was filtered using a lowpass filter with a cutoff frequency of 12 kHz. The figure shows how the method of sequential transforms removes sidelobes and sharpens the center frequency.

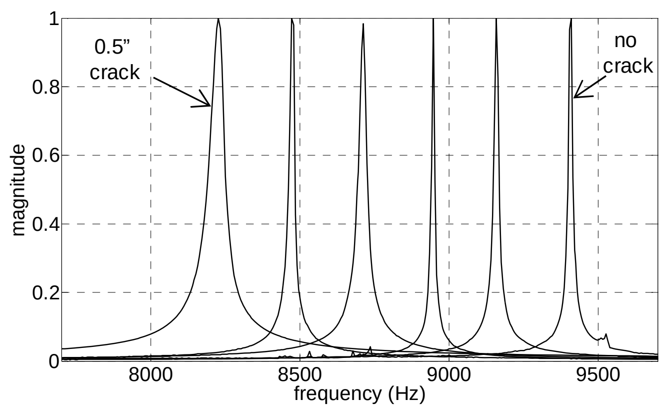

Figure 5. Second fundamental frequency for a compact tensile specimen with different crack lengths.

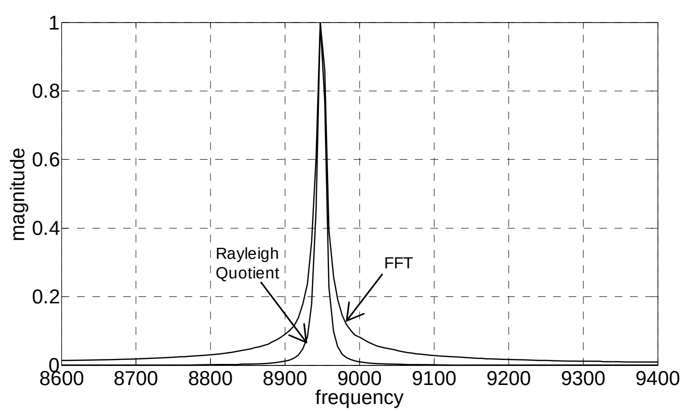

Figure 6. FFT and Rayleigh Quotient for measurements from two accelerometers mounted on the compact tensile specimen.

SUMMARY AND CONCLUSION

We have introduced a new transform method for obtaining a smoother spectrum by analyzing sequential data using an approach similar to Rayleigh's Quotient method for eigenvalue problems and super-resolution techniques. This provides a means for improving the quality of features for automated classification techniques without the need for acquiring multiple samples or using additional transducers. We have derived the approach analytically and provided a numerically efficient means to obtain the transform. The algorithm has been successfully tested on numerical data with artificially induced noise and on experimental accelerometer measurements.

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6. The MathWorks Inc., 3 Apple Hill Drive, Natick, MA 01760-2098